Winter term 2026/2027
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Exercises to Fokker–Planck equations

Sheet 1
Class: 06.11.2026

Problem 1 (Proof of Prop.1.1.3: Marginals of SDE-solution solve FPE)**.**

Prove Proposition 1.1.3.

Problem 2 (Initial values are unique)**.**

Prove: The initial value ν of a FPE-solution in the sense of Definition 1.2.3. is unique.

Problem 3 (Globally integrable solutions)**.**

Let $L_{a,b,c} = a_{ij}\partial_{ij}^2 + b_i\partial_i + c$ for Borel coefficients $a_{ij}, b_i, c : \mathbb{R}_+ \times \mathbb{R}^d \to \mathbb{R}$. Suppose $(\mu_t)_{t\geqslant 0}$ is a vaguely continuous solution to the FPE

$$\partial_t \mu_t = L_{a,b}^* \mu_t, \quad \mu_0 = \nu, \quad t \geqslant 0,$$

consisting of finite nonnegative Borel measures ν, μ_t on $\mathbb{R}^d$ such that $a_{ij}, b_i, c \in L^1([0,T]\times\mathbb{R}^d; \mu_t dt)$ for all $T > 0$.

Prove:

$$\mu_t(\mathbb{R}^d) = \nu(\mathbb{R}^d) + \int_0^t \int_{\mathbb{R}^d} c(s,x)\, d\mu_s(x) ds, \quad \forall t \geqslant 0.$$

Conclude that the solution is weakly continuous.