Winter term 2026/2027
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Exercises to Fokker–Planck equations

Sheet 4
Class: 27.11.2026

Problem 1 (Deterministic case $a = 0$)**.**

Reading exercise: Read Chapter 1.3.1 on the deterministic special case of the martingale problem and the superposition principle.

Problem 2 (Tightness and coercive functions)**.**

Recall that a family $\{\mu_i\}_{i\in I} \subseteq \mathcal{P}(S)$ of Borel probability measures on a metric space (S, d) is called *tight*, if for all $\varepsilon > 0$, there exists a compact set $K_\varepsilon \subset S$ such that $\inf_{i\in I} \mu_i(K_\varepsilon) \geqslant 1 - \varepsilon$.

A function $f : S \to [0, \infty)$ is called *coercive*, if $\{f \leqslant c\}$ is compact for all $c \geqslant 0$.

Recall that a sufficient criterion for a family $\{\mu_i\}_{i\in I} \subseteq \mathcal{P}(S)$ to be tight is the existence of a coercive function f such that

$$\sup_{i\in I} \int_S f d\mu_i < \infty.$$

Now we consider the converse statement for sequences and for $S = \mathbb{R}^d$:

Prove: For any tight sequence $(\mu_n)_{n\in\mathbb{N}} \subseteq \mathcal{P}(\mathbb{R}^d)$, there exists a coercive map $V : \mathbb{R}^d \to \mathbb{R}_+$ in $C^2(\mathbb{R}^d)$ with uniformly bounded first and second partial order derivatives such that $\sup_n \int_{\mathbb{R}^d} V \, d\mu_n < \infty$.